

b -quark decay in the collinear approximation

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Abstract

The semileptonic decay of a b -quark, $b \rightarrow c\ell\nu$, is considered in the relativistic limit where the decay products are approximately collinear. Analytic results for the double differential lepton energy distributions are given for finite charm-quark mass. Their use for the fast simulation of isolated lepton backgrounds from heavy quark decays is discussed.

Many new physics signals at hadron colliders involve isolated hard leptons as a distinguishing feature. Sequential decays of supersymmetric particles [1] and Higgs boson decays to Z , W or τ pairs [2,3] are but two examples. In all these cases the production of heavy quarks, in particular bottom and charm, and their subsequent semileptonic decay constitutes an important background. Even though lepton isolation, the requirement that little hadronic energy is deposited in the vicinity of the charged decay lepton, can reduce these heavy quark backgrounds by large factors, the sheer size of the $b\bar{b}$ or $c\bar{c}$ production cross section makes heavy flavor backgrounds dangerous [2].

For the simulation of such heavy flavor backgrounds the large suppression factors due to lepton isolation pose a special problem: large Monte Carlo samples must be generated in order to analyze the phase space distributions of the surviving events. While the full five-dimensional distribution of $b \rightarrow c\ell\nu$ decay is easily implemented in a Monte Carlo program [4], this procedure does not always generate isolated lepton events in a sufficiently fast and efficient manner.

In this brief note I describe how a fast short-cut is provided by analytic expressions for the lepton energy distributions in the laboratory frame. Leptons of sufficiently high transverse momentum can only result from the decay of very energetic b or c quarks. In turn, this implies that the parent quarks must be moving relativistically in the lab, which results in the decay products moving approximately collinear to the parent quark direction. In this relativistic limit, only the energy fractions of the decay particles, as compared to the heavy quark energy, are needed for a full description.

To be definite, consider the decay $b \rightarrow c\ell\nu$ and denote the energy fractions of the neutrino, the charged lepton and the c -quark by

$$x = \frac{E_\nu}{E_b}, \quad y = \frac{E_\ell}{E_b}, \quad z = \frac{E_c}{E_b}, \quad (1)$$

respectively. Obviously they obey the constraint $x + y + z = 1$. The smallest energy for the charm quark is reached when, in the b rest frame, it is emitted opposite to the b -quark direction, recoiling against a collinear lepton-neutrino pair:

$$z \geq r = \frac{m_c^2}{m_b^2}. \quad (2)$$

In the spectator quark model, and for unpolarized b -quarks, the double differential b -decay distribution can be determined analytically. I find

$$\frac{1}{\Gamma} \frac{d^2\Gamma}{dx dy} = \frac{2c}{f(r)} \left(c (1-x) [c + (3-c)x] + 3ry \frac{(2-c)x + c}{1-x-y} \right). \quad (3)$$

Here

$$c = \frac{1-r-x-y}{1-x-y} = 1 - \frac{r}{z}, \quad (4)$$

and $f(r)$ is the phase space suppression factor for the $b \rightarrow c\ell\nu$ decay due to the finite charm quark mass [5,6],

$$f(r) = (1-r^2)(1-8r+r^2) - 12r^2 \log r, \quad (5)$$

which is quite sizable for b -decay: $f(r) = 0.42$ for $r = 0.12$.

For $c \rightarrow s\ell\nu$ decay the $(V-A) \times (V-A)$ structure of the weak decay amplitude implies a double differential decay distribution identical to Eq. (3), but with the role of charged lepton and neutrino energy fractions interchanged, i.e. for charm decay $x = E_\ell/E_c$, $y = E_\nu/E_c$, $z = E_s/E_c \geq r = m_s^2/m_c^2$.

In the massless limit, $m_c = 0$ i.e. $r = 0$ and $c = 1$, the double differential distribution of Eq. (3) reduces to

$$\frac{1}{\Gamma_0} \frac{d^2\Gamma_0}{dx dy} = 2 (1-x) (1+2x), \quad (6)$$

which leads e.g. to the well known lepton decay distribution in $\tau \rightarrow \ell \bar{\nu}_\ell \nu_\tau$ decay [6,7]

$$\frac{1}{\Gamma_0} \frac{d\Gamma_0}{dy} = \int_0^{1-y} dx \frac{1}{\Gamma_0} \frac{d^2\Gamma_0}{dx dy} = \frac{1}{3} (1-y) (5+5y-4y^2). \quad (7)$$

The double differential decay distribution of Eq. (3) provides an adequate description of charged lepton and missing transverse momentum distributions and their correlations in typical collider physics applications. Because of its simple algebraic form, it may be folded analytically with algebraic fragmentation functions, like the Peterson fragmentation function [8] for b -quarks.

Also, the collinear limit is sufficiently simple to cast phase space limits into limits on the momentum fractions of the b -decay products, once the momentum of the parent b is known, e.g. in a Monte Carlo program.

Another application is the effect of lepton isolation on the observable charged lepton or missing transverse momentum distributions in b decays. A typical lepton isolation cut limits the energy fraction carried by the charm quark, to e.g. 10 % of the observable lepton energy, or imposes an upper limit, e.g. 5 GeV, on its transverse energy. Taking $m_b = 5.28$ GeV and $m_c = 1.87$ GeV, i.e. using the lightest meson masses in order to approximately obtain the correct kinematics for the heavy quark decays, one finds $z > r = 12.5\%$, which, at face value, excludes any events where the charm quark would carry as little as 10% of the charged lepton energy or would limit the b -quark E_T , and thereby the maximum lepton E_T to about 40 GeV when $E_{Tc} < 5$ GeV is required. However, these limits are imposed in the experiment on observed hadrons, or calorimeter response in some cone around the lepton direction. For the soft hadronic depositions inside the lepton isolation cone, non-perturbative corrections (from fragmentation or underlying event contributions) or fluctuations in the calorimeter response lead to considerable uncertainties in the true energy fraction z carried by the charm quark. The low energy tails of the calorimeter response to charm quarks are largely responsible for fake isolated lepton events. As a result, the actual z -distribution of the charm quark requires detailed simulations, except for the general statement that small values of z , close to their kinematic limit $z = r$, are strongly favored.

The double differential decay distribution derived above allows to assess the effects that z -smearing has on the observed lepton distributions. At fixed $z = 1 - x - y$ we may study the charged lepton energy distribution

$$\frac{1}{\Gamma_z} \frac{d\Gamma_z}{dy}(y) = \frac{1}{N(z)} \frac{1}{\Gamma} \frac{d^2\Gamma}{dxdy}(x = 1 - y - z, y) , \quad (8)$$

or the analogous neutrino energy distribution $1/\Gamma_z d\Gamma_z/dx$. Here $N(z)$ is a normalization factor which is obtained by direct integration of Eq. (3):

$$N(z) = \frac{1}{\Gamma} \frac{d\Gamma}{dz} = \int_0^{1-z} dx \frac{1}{\Gamma} \frac{d^2\Gamma}{dxdy}(x, y = 1 - x - z)$$

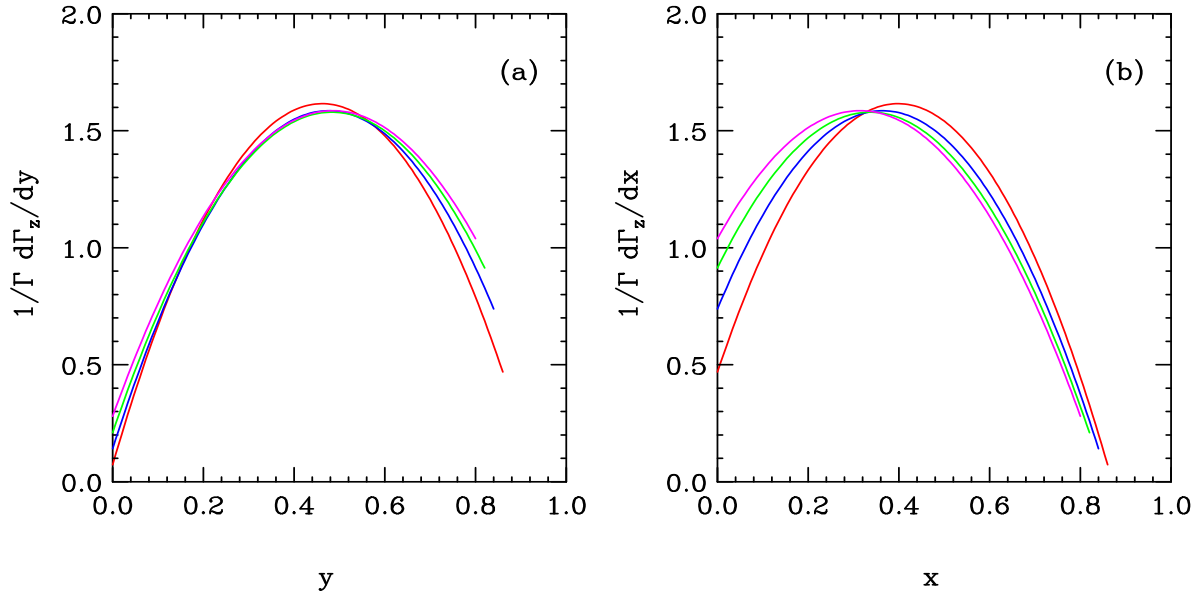


FIG. 1. Normalized energy distributions of (a) the charged lepton and (b) the neutrino in $b \rightarrow \nu \ell c$ decays in the spectator model. The individual curves correspond to fixed charm quark energy fractions $z = 0.14$ (red), 0.16 (blue), 0.18 (green), 0.2 (magenta), which are slightly above threshold, given by $z = r = m_c^2/m_b^2 = 0.12$.

$$= \frac{2}{f(r)} \left(1 - \frac{r}{z}\right)(1 - z) \left((1 - r)^2 + \frac{1}{6} \left(1 - \frac{r}{z}\right)(1 - z) \left(4z - r - 1 + 4\frac{r}{z}\right) \right). \quad (9)$$

These neutrino and charged lepton energy distributions are shown in Fig. 1. They change very little with z , in the z -range leading to isolated leptons. The largest z -dependence is found near the kinematic limits, somewhat affecting the hardest charged leptons and the softest neutrinos. The modest z -dependence of the lepton distributions implies that the precise z -distribution produced by the lepton isolation cuts is not needed for an adequate description of lepton momentum distributions.

One thus finds that in b -quark decays which lead to isolated leptons, the charged lepton and missing transverse momentum distributions, and their correlations, can be modeled quite reliably, making use of the double differential decay distribution described here. A first application in collider phenomenology appears in Ref. [9] where $b\bar{b}$ + jets backgrounds to $H \rightarrow \tau\tau$ searches are discussed: the decay distributions in the collinear approximation allow to considerably improve

Monte Carlo statistics. Similar improvements are foreseen in the simulation of heavy quark backgrounds for many new physics signals involving isolated charged leptons.

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